STEADY TWO-PHASE FLOW IN A VERTICAL TUBE SUBJECT TO COMBINED FREE AND FORCED CONVECTION

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Using the two-velocity, two-temperature model of a continuous medium, the viscous-gravitational flow of a mixture of incompressible liquid and solid particles in a vertical round tube is considered. The free-convection equations are written down on the basis of the general equation of motion and the energy equation of a two-phase medium [1, 2]. Using a finite Hankel integral transformation, a solution is constructed for the case of a linear wall-temperature distribution along the tube. The results of some practical calculations of the velocity and temperature fields over the cross section of the tube are presented, together with the dimensionless heat-transfer coefficient expressed as a function of the Rayleigh number and phase concentration. Here it is assumed that the dynamic and thermal-interaction coefficients between the phases correspond to the Stokes mode of flow for each particle, as a result of which the velocity and thermal phase lag is very small [3].

1. We shall consider the questions of flow and heat transfer in a vertical round tube of radius R at a reasonable distance from the entrance, i.e., in a region in which thermal and hydrodynamic stabilization of the flow have been achieved. In this region flow takes place parallel to the wall and the velocity remains constant along the length of the tube. The physical properties and concentrations of the phases (but not the densities) are regarded as constant. The change in the true densities as a function of temperature is (by analogy with single-phase flow [4, 5]) assumed linear, and is only taken into account in those terms of the equations of motion which express the lifting forces. The insignificant change in the velocities for the case of free convection and the negligible velocity lag between the phases allows us to exclude terms due to energy dissipation and the viscous interaction of the phases from consideration in the energy equations.

Subject to these assumptions the original system of equations [1, 2] is greatly simplified.

The z axis of the cylindrical coordinate system ρ, φ, z is directed upward along the tube axis. Allowing for the symmetry of the flow and heat transfer, the equation for the gravitational convection of the two-phase medium (subject to the foregoing assumption) then becomes

$$\mu_i\varphi_i\left(\frac{\partial^2 W_i}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial W_i}{\partial\rho}\right)-(-1)^i k_0(W_2-W_1)-\varphi_i\frac{\partial p_i}{\partial z}-\varphi_i\rho_i g[1-\beta_i(T_i-T_w)]=0;$$

$$W_i\varphi_i\rho_i c_{pi}\frac{\partial T_i}{\partial z}=\varphi_i\lambda_i\left(\frac{\partial^2 T_i}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial T_i}{\partial\rho}\right)-(-1)^i\alpha_0(T_2-T_1), \tag{1.1}$$

$$\varphi_1+\varphi_2=1, \quad (i=1,2).$$

Here W_i is the velocity, T_i is the temperature, p_i is the pressure, φ_i is the concentration, ρ_i is the true density for a wall temperature of T_w; μ_i, β_i, c_{pi}, λ_i are the dynamic viscosity, volume expansion, specific heat at constant pressure, and thermal conductivity of phase,

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i, k_0 and α_0 are the coefficients of dynamic and thermal interaction between the phases, g is the gravitational acceleration, and i=1, 2, respectively, denote the first and second phases.

In the region corresponding to thermal stabilization of the flow, the temperatures reckoned relative to the wall temperature remain constant along the length of the tube:

$$T_i - T_w = \vartheta_i(\rho) \quad (i=1,2), \tag{1.2}$$

while the longitudinal temperature gradients at every point of the flow, including the wall, also remain constant:

$$\frac{\partial T_1}{\partial z} = \frac{\partial T_2}{\partial z}\frac{\partial T_w}{\partial z} = A = \text{const.} \tag{1.3}$$

For convenience of subsequent calculations we write the system (1.1) in dimensionless form, taking account of (1.2) and (1.3):

$$\begin{gathered}
\varphi_1\left(\frac{d^2U_1}{dr^2}+\frac{1}{r}\frac{dU_1}{dr}\right)+k(U_2-U_1)+\varphi_1\,\mathrm{Ra}\,\theta_1=\varphi_1P_1;\\
\varphi_2\delta_1\left(\frac{d^2U_2}{dr^2}+\frac{1}{r}\frac{dU_2}{dr}\right)+k(U_1-U_2)+\varphi_2\delta_2\,\mathrm{Ra}\,\theta_2=\varphi_2P_2;\\
\varphi_1U_1=\varphi_1\left(\frac{d^2\theta_1}{dr^2}+\frac{1}{r}\frac{d\theta_1}{dr}\right)+\alpha(\theta_2-\theta_1);\\
\varphi_2\delta_3U_2=\varphi_2\delta_4\left(\frac{d^2\theta_2}{dr^2}+\frac{1}{r}\frac{d\theta_2}{dr}\right)+\alpha(\theta_1-\theta_2),
\end{gathered} \tag{1.4}$$

in which we have introduced the following notation:

$$\begin{gathered}
U_i = W_i\frac{\mu_1}{\rho_1 gR^2};\quad \theta_i=\vartheta_i\frac{\mu_1\lambda_1}{AgR^4\rho_1^2c_{p1}},\quad (i=1,2),\quad r=\frac{\rho}{R};\\
k=\frac{k_0R^2}{\mu_1};\quad \alpha=\frac{\alpha_0R^2}{\lambda_1};\quad \delta_1=\frac{\mu_2}{\mu_1};\quad \delta_2=\frac{\rho_2\beta_2}{\rho_1\beta_1};\\
\delta_3=\frac{\rho_2c_{p2}}{\rho_1c_{p1}};\quad \delta_4=\frac{\lambda_2}{\lambda_1};\quad P_1=1+\frac{1}{g\rho_1}\frac{dp_1}{dz};\quad P_2=\frac{\rho_2}{\rho_1}+\frac{1}{g\rho_1}\frac{dp_2}{dz};
\end{gathered}$$

$\mathrm{Ra} = (AgR^4\beta_1/\mu_1\lambda_1)\,\rho_1^2c_{p1}$ is the Rayleigh number of the first phase.

The problem thus reduces to the solution of a system of equations (1.4) subject to the boundary conditions

$$\begin{gathered}
U_i=\theta_i=0\ (i=1,\ 2),\ r=1;\\
\frac{dU_i}{dr}=\frac{d\theta_i}{dr}=0\quad (i=1,2),\quad r=0.
\end{gathered} \tag{1.5}$$

After using a finite Rankel integral transformation, the solution of the boundary problem (1.4), (1.5) is obtained in the form

$$\begin{gathered}
U_i = 2\sum_{n=1}^{\infty}U_{in}\frac{\mathcal{J}_0(H_nr)}{H_n\mathcal{J}_1(H_n)},\quad (i=1,2);\\
\theta_i = 2\sum_{n=1}^{\infty}\theta_{in}\frac{\mathcal{J}_0(H_nr)}{H_n\mathcal{J}_1(H_n)},\quad (i=1,2);\\
\theta_{1n}=\frac{\varphi_1}{D_n}[\varphi_1P_1(\varphi_2B_{2n}+C)+\varphi_2P_2(\varphi_2B_{1n}+C)];\\
\theta_{2n}=\frac{\varphi_2}{D_n}[\varphi_1P_1(\varphi_2A_{2n}+C)+\varphi_2P_2(\varphi_2A_{1n}+C)];
\end{gathered}$$

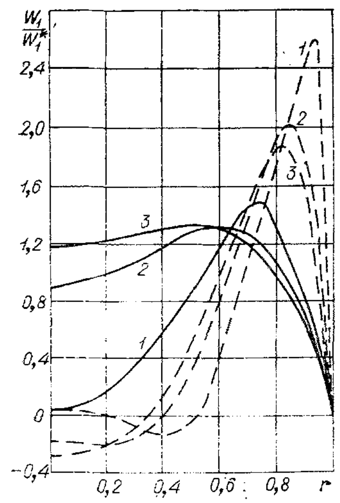

Fig. 1

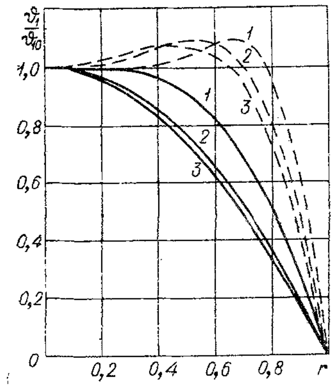

Fig. 2

$$U_{1n} = \frac{\alpha\varphi_2}{D_n}[\varphi_1 P_1 (A_{2n} - B_{2n}) + \varphi_2 P_2 (A_{1n} - B_{1n}) - H_n^2 \theta_{1n};$$

$$U_{2n} = \frac{\alpha\varphi_1}{\delta_3 D_n}[\varphi_1 P_1 (B_{2n} - A_{2n}) + \varphi_2 P_2 (B_{1n} - A_{1n})] - \frac{\delta_4 H_n^2}{\delta_3}\theta_{2n}, \qquad (1.6)$$

where the constants A_{in}, B_{in} ($i = 1, 2$), c, D_n are given by the expressions

$$A_{1n} = (\varphi_1 H_n^2 + k)(\varphi_1 H_n^2 + \alpha) + \varphi_1^2 \mathrm{Ra};$$

$$A_{2n} = (\varphi_1 H_n^2 + \alpha) k + \frac{\varphi_1 \delta_1 \alpha H_n^2}{\delta_3};$$

$$B_{1n} = (\varphi_1 H_n^2 + k)\alpha + \frac{\varphi_1 k H_n^2 \delta_4}{\delta_3};$$

$$B_{2n} = k\alpha + \frac{\varphi_1 H_n^2}{\delta_3}(\delta_4 k + \delta_1 \alpha) + \frac{\varphi_1 \varphi_2}{\delta_3}(\delta_1 \delta_4 H_n^4 + \delta_2 \delta_3 \mathrm{Ra}), C = \frac{k\alpha\varphi_1}{\delta_3};$$

$$D_n = \varphi_2 (A_{1n} B_{2n} - A_{2n} B_{1n}) + C (A_{1n} - A_{2n} + B_{2n} - B_{1n}),$$

($\mathcal{J}_0$ and $\mathcal{J}_1$ are Bessel functions of the first kind and the zero and first orders; H_n are roots of the equation $\mathcal{J}_0(H)=0$) .

If the physical properties of the phases are identical, i.e., $\delta_i=1$ ($i=1, 2, 3, 4$), $P_1=P_2=P$ or the concentration of the second phase is equal to zero, Eq. (1.6) yields the solution to the problem of interaction between free and forced convection in a single-phase medium [4, 5]:

$$U_1 = U_2 = -2P \sum_{n=1}^{\infty} \frac{1}{\mathrm{Ra} + H_n^4} \frac{\mathcal{J}_0(H_n r)}{H_n \mathcal{J}_1(H_n)};$$

$$\theta_1 = \theta_2 = 2P \sum_{n=1}^{\infty} \frac{H_n}{\mathrm{Ra} + H_n^4} \frac{\mathcal{J}_0(H_n r)}{\mathcal{J}_1(H_n)}.$$

2. Using Eqs. (1.6) we executed calculations for the following constants: $\delta_1=11.9$; $\delta_2=1.3$; $\delta_3=0.4$; $\delta_4=0.17$; $P_1/P_2=2$. The coefficients k_0 and α_0 were taken in the form [1, 6]

$$k_0 = \frac{9}{2}\frac{\mu_1}{l^2}\varphi_2; \qquad \alpha_0 = 3\frac{\lambda_1}{l^2}\varphi_2,$$

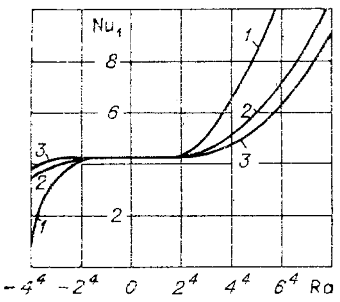

Fig. 3

where l is the radius of the particles forming the second phase. The dimensionless complexes k and α are thus calculated from the formulas

$$k = \frac{9}{2}\left(\frac{R}{l}\right)^2 \varphi_2; \qquad \alpha = 3\left(\frac{R}{l}\right)^2 \varphi_2.$$

Figures 1 and 2 show the velocity and temperature profiles of the first phase for $R/l^2 = 10^2$, respectively referred to the velocity W^*_1 averaged over the cross section and the temperature ϑ_{10} on the tube axis. The continuous lines correspond to Ra = 625 and the broken lines to Ra = 10^4. Curves 1-3 correspond to $\varphi_2=0$; 0.2; 0.4.

We see from Figs. 1 and 2 that for large positive Ra numbers the principal change in temperature and hence the main rise in velocity takes place in the region close to the wall. With increasing impurity concentration the rise in flow velocity and temperature takes place more slowly than in a single-phase medium for the same Ra number.

It follows from calculations carried out for various values of Ra that the velocity in the core of the flow falls with increasing Ra, the more rapidly the smaller the concentration of the second phase. Whereas, for a single-phase medium, concavity of the velocity profile close to the axis begins at Ra = 64.14 [5], for a flow containing foreign particles concavity begins at a higher value Ra > 64.14; further development leads to a change in the direction of flow in the core. For negative Ra values (corresponding to heating from the bottom) the flow velocity increases in the middle of the tube and falls close to the wall with Ra. This leads to the development of reverse flow at the wall. With increasing concentration of the second phase and the same value of Ra < 0 the velocity in the core of the flow diminishes, whereas it increases close to the wall. For Ra values close to zero, quite independently of the concentration of the second phase, the velocity and temperature distributions over the cross section of the tube differ little from the corresponding single-phase flow distributions.

Figure 3 shows the Nusselt number Nu_1 as a function of Ra and concentration φ_2, calculated for $R/l = 10^2$. Curves 1-3 correspond to $\varphi_2=0$; 0.2; 0.4. In this calculation we used the average mass temperature of the first phase $\theta_1^* = 2\int_0^1 \theta_1 U_1/U_1^* \, r dr$, i.e., the Nu_1 number was calculated from the equation

$$\mathrm{Nu}_1 = -\frac{2}{\theta_1^*}\frac{d\theta_1}{dr}\Big|_{r=1} = 4\,\frac{\sum\limits_{n=1}^{\infty}\theta_{1n}\sum\limits_{n=1}^{\infty}\frac{1}{H_n^2}U_{1n}}{\sum\limits_{n=1}^{\infty}\frac{1}{H_n^2}\theta_{1n}U_{1n}}.$$

We see from Fig. 3 that for $-4^4 < \mathrm{Ra} < 0$ the Nu_1 number rises with increasing concentration, while for Ra>0 the reverse is the case. With increasing Ra the Nu_1 number increases, independently of the concentration; as Ra→0 it approaches a constant value of 4.36, which is characteristic of the purely forced convection of a single-phase medium.

In the case of homogeneous flow reverse flow starts at the walls at a value of Ra = −104.9, while for $\mathrm{Ra} \leqslant -168$ stability is infringed and the character of the heat transfer alters; for Ra < −250 the flow becomes turbulent [5]. Calculations show that for a flow containing foreign particles reverse flow starts at the wall when R < −104.9 (for example, for $\varphi_2=0.2$ and Ra = −187 reverse flow at the wall is still absent), while the change in the Nusselt number remains as before, even for Ra < −250. We may deduce from this that the transition from laminar two-phase to turbulent flow sets in at Ra numbers smaller than those corresponding to a homogeneous flow.

In Figs. 1-3 curves 1 coincide with the corresponding curves for a single-phase flow given in [5].

Calculations carried out for $R/l=10^4;\ 10^5$ show that, for the same concentration of the second phase, the dimensions of the particles do not make any major contribution to the velocity and temperature distributions. In all cases the difference between these quantities for the first and second phase is negligible, i.e., the assumption of Stokes flow around the particles is satisfied. Thus the foregoing solutions are valid for flows containing foreign particles in tubes with a ratio of $R/l > 10^2$.

LITERATURE CITED

1. M. E. Deich and G. A. Filippov, Gasdynamics of Two-Phase Media [in Russian], Énergiya, Moscow (1968).
2. S. Sou, Hydrodynamics of Multiphase Systems [Russian translation], Mir, Moscow (1971).
3. A. N. Kraiko and R. A. Tkalenko, "Solution of the direct problem in the theory of a Laval nozzle for a two-phase mixture with a slight particle lag," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1973).
4. G. A. Ostroumov, Free Convection under the Conditions of an Internal Problem [in Russian], GITTL, Moscow–Leningrad (1952).
5. B. S. Petukhov, Heat Transfer and Resistance in the Laminar low of Liquid in Tubes [in Russian], Énergiya, Moscow (1967).
6. G. A. Saltanov and R. A. Tkalenko, "Flow of a supersonic two-phase stream around a wedge," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2 (1972).